The production of uniform shear flow in a wind tunnel

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Summary

A nearly uniform shear flow was obtained in the working section of a wind tunnel by inserting a grid of parallel rods with varying spacing.

The function of such a grid is to impose a resistance to the flow, so graded across the working section as to produce a linear variation in the total pressure at large distances downstream without introducing an appreciable gradient in static pressure near the grid. A method of calculating a suitable arrangement of the rods is described. Although this method is strictly applicable only to weakly sheared flows, an experiment made with a grid designed for a shear parameter as large as 0·45 gave results in close agreement with the theory. There was no evidence from the experiment of any large-scale secondary flow accompanying the shear—a danger inherent in an empirical attempt to grade the resistance of the grid—nor was any tendency observed for the shear to decay with increasing distance from the grid.

1. Introduction

A simple method of obtaining a two-dimensional shear flow $u(y)$ with vorticity ω_z in a wind tunnel is to insert near the beginning of the working section a grid of parallel rods in which the distance between adjacent rods varies in the y-direction, as sketched in figure 1. The effect of such an arrangement is clearly to produce a grading in the resistance of the grid which in turn gives rise to a non-uniform (irrotational) flow upstream of the grid and a non-uniform distribution of total pressure downstream of it. If it were possible to select a resistance grading such that the gradient of total pressure far downstream of the grid were constant, the vorticity ω_z would also be constant.

In principle it should not be difficult to devise empirically a grid which satisfies approximately the above condition on total pressure gradient (a remark on the feasibility of realizing it in practice is made in § 5), but such an approach carries with it the danger of introducing secondary vorticity components ω_x, ω_y since the variation of total pressure near the downstream face of the grid might include large static pressure gradients.

The method of grid design described in this paper has as its purpose the calculation of a resistance grading corresponding to a constant total pressure gradient far downstream together with a predictable and, if possible, small static pressure gradient near the grid, and is applicable to weakly-sheared flows in the sense that the maximum change in velocity across the wind tunnel is small compared with the mean velocity.

On the basis of the method, a particular grid was designed and tested in a wind tunnel with a working section 20 in. square at a mean speed, near the tunnel axis, of approximately 75 ft. sec^{-1}. Static pressure and Pitot pressure traverses across the working section at a number of stations downstream of the grid revealed, over a large part of the area, an approximately linear velocity distribution, $u(y)$, which was almost invariant with both x and z. Furthermore, the static pressure was found to be sensibly constant over planes parallel to the grid at distances from it of one tunnel width and greater; the region in the immediate vicinity of the grid could not be explored with a conventional static probe for reasons explained in § 5.

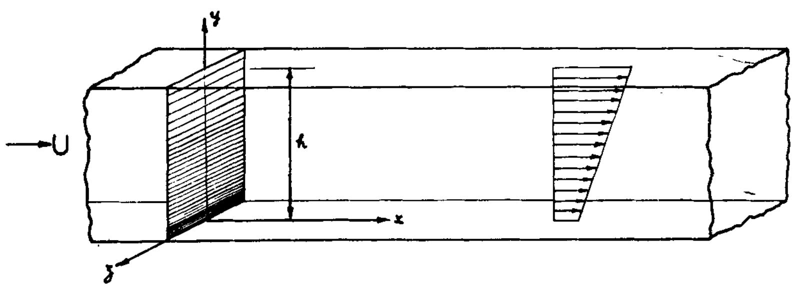

Figure 1. Arrangement of the grid and coordinate system.

One of the objects of producing a shear flow on a large scale is to examine the flow about bodies in a stream of uniform vorticity, and thereby to undertake the experimental counterpart of theoretical investigations made principally by Hawthorne & Martin (1955) and Lighthill (1956). It is hoped to report later on the results of these experiments.

2. Method of grid design

The wind tunnel may be treated as a long channel with walls $y = 0$, $y = h$, in which a grid is placed in the plane $x = 0$. At great distances upstream the velocity is uniform and has the value U; far downstream the velocity is again parallel to the walls and its magnitude is given by

$$u = U + \lambda(y - \tfrac{1}{2}h). \tag{1}$$

We suppose the fluid is inviscid and that $\lambda h/U$ is so small that the departure of any streamline from a straight line is correspondingly small. Accordingly, the stream function may be written

$$\begin{aligned} \psi &= Uy + \psi', & x < 0, \\ &= Uy + \tfrac{1}{2}\lambda(y^2 - hy) + \psi', & x > 0, \end{aligned} \tag{2}$$

where ψ', which is everywhere small compared with Uh and tends to zero as $x \to \pm\infty$, satisfies $\nabla^2\psi' = 0$. Appropriate solutions satisfying the boundary conditions

$$\frac{\partial\psi'(x,0)}{\partial x} = \frac{\partial\psi'(x,h)}{\partial x} = 0$$

for all x are

$$\begin{aligned}\frac{\psi'}{Uh} &= \sum_{n=1}^{\infty} A_n\, e^{n\pi x/h} \sin(n\pi y/h), \qquad x < 0,\\ &= \sum_{n=1}^{\infty} B_n\, e^{-n\pi x/h} \sin(n\pi y/h), \qquad x > 0.\end{aligned} \tag{3}$$

The presence of the grid requires that (i) $\partial\psi/\partial y$ is continuous through $x = 0$, (ii) the velocity component $\partial\psi/\partial x$ obeys a certain refraction condition across $x = 0$, and (iii) the difference between the total pressures on a given streamline for large positive and negative values of x must be equal to the resistance per unit area imposed by the grid. To these may be added the condition that the static pressures far upstream and far downstream of the grid are independent of y.

Condition (i) leads to

$$\pi \sum_{n=1}^{\infty} nA_n \cos(n\pi y/h) - \pi \sum_{n=1}^{\infty} nB_n \cos(n\pi y/h) = \left(\frac{y}{h} - \frac{1}{2}\right)\frac{\lambda h}{U},$$

or, setting

$$\tfrac{1}{2}y(y-h) = h^2 \sum_{n=1}^{\infty} C_n \sin(n\pi y/h),$$

$$A_n - B_n = C_n \lambda h/U. \tag{4}$$

The refraction condition (ii) requires the y-component of the velocity to change by a factor α in passing through the grid. α for a grid of uniform resistance is known to be $1{\cdot}1(1+K)^{-1/2}$, where K is the resistance coefficient (Taylor & Batchelor 1949), and it is unlikely that this value will be seriously in error for a grid with slightly graded resistance provided that K is given its local value. It follows that, if the resistance grading is represented by $K = K_0[1+\epsilon(y)]$, such that ϵ is small and $\int_0^h \epsilon\, dy = 0$,

$$\alpha = \frac{1{\cdot}1}{(1+K_0)^{1/2}}\left[1 - \frac{K_0\,\epsilon}{2(1+K_0)}\right] + O(\epsilon^2). \tag{5}$$

With α given by (5), the refraction condition is

$$\sum_{n=1}^{\infty} nB_n \sin(n\pi y/h) + (a + b\epsilon) \sum_{n=1}^{\infty} nA_n \sin(n\pi y/h) = 0,$$

which, noting that A_n and B_n are $O(\lambda h/U)$, reduces to

$$B_n + aA_n = 0 + O(\lambda h\epsilon/U), \tag{6}$$

where $a = 1{\cdot}1(1+K_0)^{-1/2}$.

Finally, we have to consider the change in total pressure along a streamline. According to the assumption of an inviscid fluid, the total pressure remains constant along streamlines upstream and downstream of the grid but, at the grid itself, there is a decrease in total pressure amounting to $K\frac{1}{2}\rho[u(0)]^2$. Hence,

$$p_0+\tfrac{1}{2}\rho U^2-p_1-\tfrac{1}{2}\rho[U+\lambda(y_1-\tfrac{1}{2}h)]^2 = K\tfrac{1}{2}\rho[u(0)]^2, \tag{7}$$

where p_0 and p_1 are respectively the static pressures far upstream and far downstream of the grid. The streamline which has the ordinate y_1 for large x meets the grid at $y = y_0$, and the relation between y_0 and y_1 follows from (2);

$$y_1 = y_0+\psi'(0,y_0)/U+O(\lambda\psi'/U^2). \tag{8}$$

Equation (8) implies that, to the first order in $\lambda h/U$ and ψ'/Uh, the x-component of the velocity at the grid is given by

$$u(0) = U+\lambda(y_0-\tfrac{1}{2}h)+\frac{\partial\psi'(0,y_0)}{\partial y}, \tag{9}$$

which, together with (7), leads to

$$\frac{p_0-p_1}{\frac{1}{2}\rho U^2} = K_0(1+\epsilon)+2\frac{\lambda h}{U}(1+K_0)\left(\frac{y_0}{h}-\frac{1}{2}\right)+\frac{2K_0}{U}\frac{\partial\psi'(0,y_0)}{\partial y}. \tag{10}$$

Since (p_0-p_1) is independent of y, (10) resolves into

$$\frac{p_0-p_1}{\frac{1}{2}\rho U^2} = K_0 \tag{11}$$

and

$$K_0\epsilon+2\frac{\lambda h}{U}(1+K_0)\left(\frac{y_0}{h}-\frac{1}{2}\right)+2\frac{K_0}{U}\frac{\partial\psi'(0,y_0)}{\partial y} = 0. \tag{12}$$

The distribution $\epsilon(y_0)$ follows directly from (12) if we note from (3), (4) and (7) that

$$\frac{\partial\psi'(0,y_0)}{\partial y} = -\pi\lambda h^2\frac{a}{(1+a)}\sum_{n=1}^{\infty} nC_n\cos(n\pi y/h) = -\lambda h\frac{a}{(1+a)}\left(\frac{y_0}{h}-\frac{1}{2}\right); \tag{13}$$

hence,

$$\epsilon(y_0) = -2\frac{\lambda h}{U}\left(\frac{1}{K_0}+\frac{1}{1+a}\right)\left(\frac{y_0}{h}-\frac{1}{2}\right). \tag{14}$$

This result could have been arrived at without introducing the Fourier series, but the series representation (3) is useful for calculating streamlines and pressures away from the grid.

3. Pressures on the faces of the grid

Equations (9) and (13) demonstrate that, of the ultimate perturbation $\lambda(y-\frac{1}{2}h)$ to the velocity in the channel, a fraction $1/(1+a)$ appears at the grid and is produced irrotationally in the flow upstream. Clearly, as a approaches zero, a condition which could be realized by making K_0 very large, a progressively greater proportion of the velocity adjustment is achieved before the flow reaches the grid; consequently the static pressure on the downstream face of the grid must tend to a constant value consistent with a uniform distribution of vorticity ω_z. (The static pressure on the *upstream* face will, of course, not be constant and we shall refer to its effect presently). Whilst the implication of this argument is clear enough as a criterion for avoiding the development of secondary circulation in the flow downstream of the grid, it is not very helpful in practice because there is always a limit to the value of the resistance beyond which a useful performance cannot be obtained from the wind tunnel in which the grid is placed.

To explore the possibility of a compromise between constancy of static pressure in the plane $x = +0$ and a tolerably small value of K_0—approximately 1, say—note that the pressure on the downstream face of the grid is given by

$$-C_p(+0,y_0) = \frac{p_0 - p(+0,y_0)}{\frac{1}{2}\rho U^2} = K_0 - \left(\frac{\lambda h}{U}\right)\frac{2a}{(1+a)}\left(\frac{y_0}{h} - \frac{1}{2}\right) + O\left(\frac{\lambda^2 h^2}{U^2}\right). \tag{15}$$

Although (15) shows that the pressure coefficient departs from a constant value only by $O(\lambda h/U)$ when K_0, and therefore a, are $O(1)$, it cannot be inferred immediately that such a pressure variation is acceptable unless the associated secondary vorticity, ω_x, is of a still smaller order of magnitude; for the primary (non-dimensional) vorticity $\omega_z h/U$ induced by the grid is itself only $O(\lambda h/U)$.

The development of secondary vorticity arises from the action of the pressure gradient $\partial p/\partial y$ on the fluid in the boundary layers on the tunnel walls at $z = \pm\frac{1}{2}h$. Such fluid, on account of its reduced velocity in the x-direction, more readily acquires a motion in the y-direction in response to the pressure gradient than the faster-moving fluid in the main stream. Once a cross-flow is well-established in the boundary layers a compensating flow must appear in the main stream in order to preserve continuity, and it follows that, if the maximum cross-flow velocity in the boundary layers is v', the secondary vorticity in the main stream will be $O(v'\delta/h^2)$, at most, where δ is the boundary layer thickness.

Now the maximum value of the pressure gradient $\partial p/\partial y$ is, from (15), $\rho U\lambda a/(1+a)$ on the downstream face of the grid, and it falls off with x like $e^{-\pi x/h}$. On the assumption of laminar boundary layers on the walls of the tunnel (it is known from experiments with curved pipes that the secondary flow in these circumstances is more powerful than when the flow is turbulent), a rough argument based on consideration of the balance between the flux of boundary-layer momentum in the y-direction and the combined forces due to the pressure gradient and skin friction indicates that the maximum value of v' attained some small distance downstream of the plane of the grid is $O(\lambda h)$. It follows that ω_x is $O(\lambda\delta/h)$ at most and, owing to the presence of the small factor δ/h, is of an order of magnitude smaller than ω_z. Since the boundary layers downstream of a grid are certain to be turbulent in any apparatus of useful dimensions, the mean resistance may safely be confined to values near 1.

Turning our attention to the flow upstream of the grid, we notice that any attempt to ameliorate the pressure gradient at $x > 0$ by increasing K_0 will result in an increased pressure gradient at $x < 0$, for the maximum values of $\partial p/\partial y$, which occur on the faces of the grid, are related by

$$\left(\frac{\partial C_p}{\partial y}\right)_{x=-0} = \frac{1}{a}\left(\frac{\partial C_p}{\partial y}\right)_{x=+0},$$

and so the danger of inducing a secondary circulation is merely transferred from $x > 0$ to the region $x < 0$. We have however already rejected the use of large K_0 on the grounds of wind tunnel performance, and it remains to consider the suitability of values of K_0 near 1. The problem can be disposed of quickly by appealing to the arguments of the preceding paragraph. The conclusions reached there hold equally well for the region $x < 0$, with the reservation that the increased severity of the transverse pressure gradient in the flow approaching the grid ($a < 1$ for $K_0 \doteqdot 1$) will be offset by the smaller boundary layer thickness on the vertical walls and by the attenuating action of the grid on any secondary vorticity that happened to develop.

4. Resistance grading

At large Reynolds numbers the resistance of a grid of rods in a uniform stream can be predicted with good accuracy if the drag coefficient based on the blocked area and the average interstitial velocity is taken to be 1 (Wieghardt 1953). For our present purpose it will be assumed permissible to adopt the same procedure for each portion of the grid, with the possible exception of places where the spacing changes rapidly with y, since the scale of variation in the y-direction of the velocity through the plane of the grid, $u(0)$, is much larger than the greatest spacing between adjacent rods. Accordingly, $K_0(1+\epsilon) = \xi(1-\xi)^{-2}$, where $\xi = d/s$; d is the diameter of each rod and the spacing is $s(y)$. With ϵ given by (14),

$$\frac{\xi}{(1-\xi)^2} = K_0\left[1-2\frac{\lambda h}{U}\left(\frac{1}{K_0}+\frac{1}{1+a}\right)\left(\frac{y}{h}-\frac{1}{2}\right)\right]. \tag{16}$$

It will be observed from (16) that the choice of K_0 is not entirely arbitrary, since for each value of K_0 there is a maximum value of $\lambda h/U$ which can be reached with a physically-realizable grid. This maximum, which is given in the following table, is governed by the condition that $\xi > 0$ at $y = h$.

K_0	0·2	0·4	0·6	0·8	1·0
$(\lambda h/U)_{max}$	0·18	0·33	0·45	0·56	0·64

5. Experiment

A grid composed of parallel rods of 0·125 in. diameter was constructed according to (16) with $\lambda h/U = 0·45$ and $K_0 = 1·15$. A conservative choice of K_0, as compared with the values shown in the table above, was considered desirable in order to avoid unduly rapid rates of change of the rod spacing with y near $y/h = 1$. The appropriate values of ξ are shown in figure 2, which also contains the values measured on the centre-line, $z = 0$, of the grid after it was built.

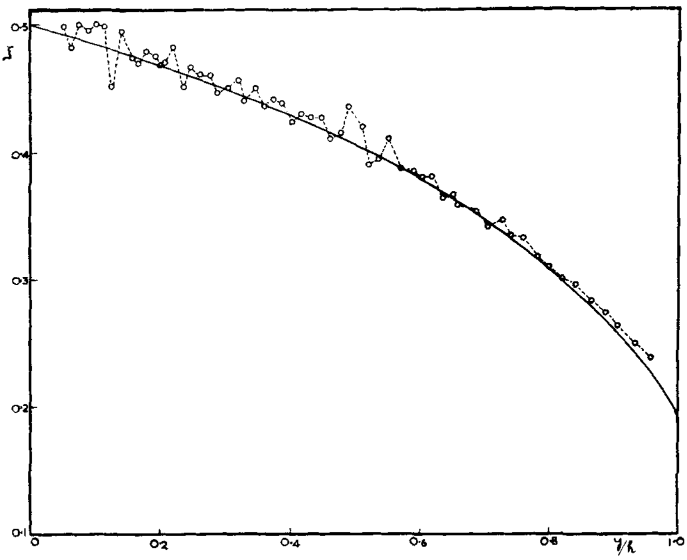

Figure 2. Spacing of the rods; ——— design values of ξ, – – o – – o – – measured values on the grid centre-line.

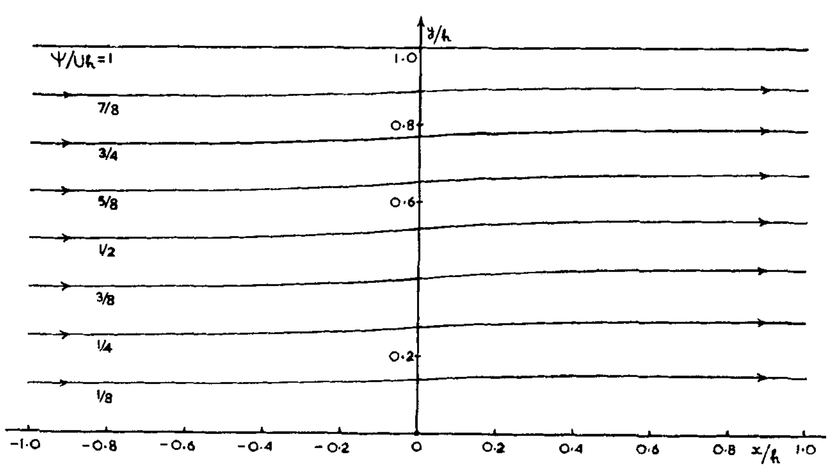

Figure 3. Calculated streamline pattern for $\lambda h/U = 0{\cdot}45$ and $K_0 = 1{\cdot}15$.

The grid was placed at the entrance to the working section, 20 in. square, of the low-turbulence wind tunnel at the Mechanics of Fluids Laboratory. Traverses across the working section in planes parallel to the grid at a number of distances from it were made with Pitot and static tubes of 1 mm outside diameter at a mean windspeed U of roughly 75 ft. sec^{-1}.

No variation in static pressure could be detected from the traverses, which were confined to distances greater than approximately one tunnel height from the grid. The grid could not be approached much more closely with a conventional static probe owing to an appreciable inclination and curvature of the streamlines, the predicted pattern of which is shown in figure 3.

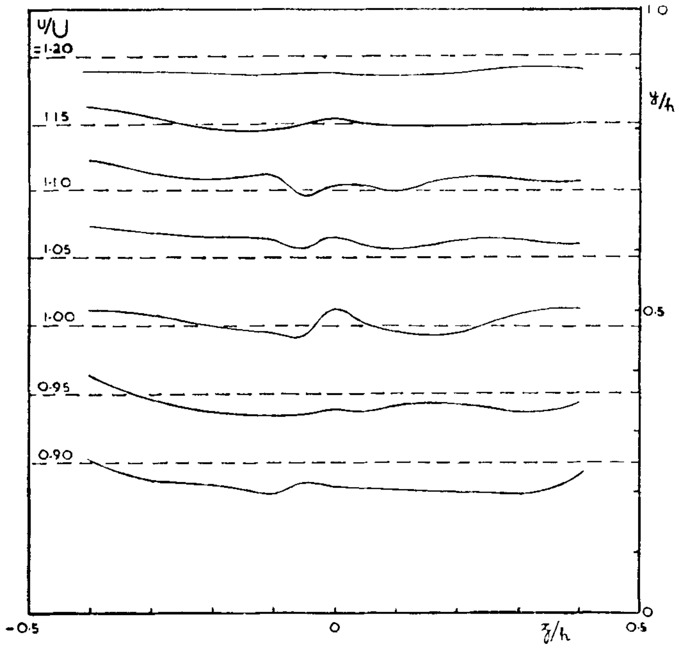

Figure 4. Velocity contours in the plane $x/h = 3$; ——— measured, - - - - - calculated.

The contours of u/U as deduced from the Pitot and static pressure traverses in a plane parallel to the grid and 5 ft. ($x/h = 3$) downstream of it are shown in figure 4. The absence of any systematic departure from lines parallel to the z-axis, together with the previously-mentioned fact that the static pressure is constant over the region, suggests that no secondary flow was present on a large scale.

The profiles of u/U in the plane of symmetry of the tunnel, $y = 0$, at distances of 2, 5 and 7 ft. from the grid ($x/h = 1{\cdot}2,\ 3{\cdot}0,\ 4{\cdot}2$) are presented in figure 5. The most striking feature of this figure is the close agreement between the measured velocity distribution and that predicted by the theory: an agreement which is remarkable in view of the fact that the grid was designed for a value of $\lambda h/U$ of 0·45, which could hardly be accepted as small enough to come within the scope of a linear theory. To be sure, the presumed effects of neglecting squares and higher powers of $\lambda h/U$ can be interpreted in the experimental velocity profile as a deficit in the average rate of shear together with a small departure from linearity, although the latter effect is equally likely to be due to an inadequacy in the method of relating the spacing of the rods to the local resistance. (The curvature of the velocity profile in figure 5 is most marked in the interval $0{\cdot}70 < y/h < 0{\cdot}95$, where according to figure 2, ξ changes rapidly with y/h.)

The small-scale irregularities in the velocity profile, which were found to be repeatable in the experiments, can be attributed to inaccuracies in the manufacture of the grid, as displayed in figure 2, and a detailed examination of the measurements (not all of which, for the sake of clarity, are reproduced in figure 5) suggested that the small amplitude waviness

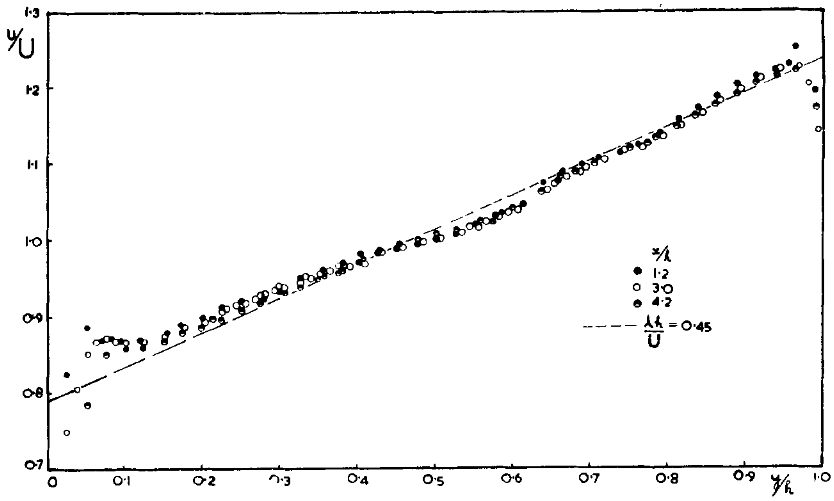

Figure 5. Velocity profiles measured in the plane of symmetry of the tunnel.

of the velocity profile in the region $y/h < 0{\cdot}5$ could be associated with the corresponding abrupt fluctuations in ξ. In particular, the pronounced dip in the velocity profile near $y/h = 0{\cdot}5$ might be caused by the error in ξ at roughly the same value of y/h; for it is easily shown that the change in u/U brought about by a small change in ξ in the interval

$$y - \tfrac{1}{2}\Delta y < y < y + \tfrac{1}{2}\Delta y$$

is given by*

$$\Delta\xi = -\frac{1}{K_0^2}\left(1 + \frac{K_0}{1+a}\right)\left(\frac{1+2K_0}{(1+4K_0)^{1/2}} - 1\right)\frac{\Delta u}{U}, \tag{17}$$

provided that outside the interval ξ is altered by a constant amount so as to preserve the condition $\int_0^h \epsilon\, dy = 0$. In the present case, with $K_0 = 1{\cdot}15$, $a = 0{\cdot}75$ and $\Delta\xi \doteqdot 0{\cdot}015$, (17) gives $\Delta u/U \doteqdot -0{\cdot}03$ which is near to the

* This relation offers the possibility of an iterative method of designing the grid, by adjusting ξ to compensate for the differences between the required velocity profile and that observed in an initial experiment. Such complication was not considered necessary for the case considered here, although in choosing a design value of $\lambda h/U$ as large as $0{\cdot}45$ we were anticipating an iterative approach.

observed amplitude of the dip ($\Delta u/U \doteqdot -0{\cdot}02$). Closer agreement could not be expected because the error $\Delta\xi$, being caused by a sag in some of the rods rather than a bodily displacement, is not independent of z, and the resulting distortion of the flow from that of nearly-uniform shear is essentially three-dimensional and must resemble the effect of a wake embedded in the stream. Indeed, a comparison between the observed velocity profiles at $z/h = 0$ and $z/h = -0{\cdot}2$ displayed in figure 6 for $x/h = 3$ shows that the dip had almost disappeared at $z/h = -0{\cdot}2$ (4 in. from the central plane of the tunnel).

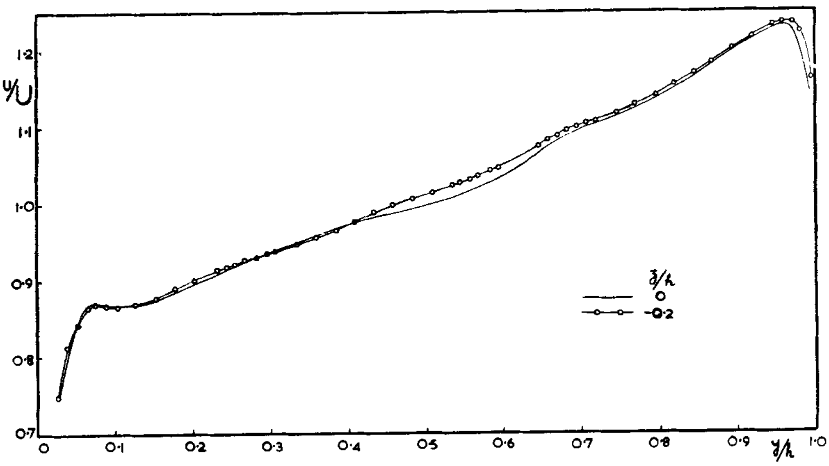

Figure 6. Velocity profiles for $z = 0$ and $z/h = -0{\cdot}2$ measured at $x/h = 3$.

The above calculation provides a stricture on the remark made in the Introduction that it might not be difficult (in principle) to devise empirically the correct spacing between the rods, because the misalignment that gave rise to the error 0·015 in ξ amounted only to 0·02 in. The chance of adjusting by experiment the positions of 62 rods, as used here, to an accuracy significantly greater than 0·02 in. seems very remote!

The velocity profiles of figure 5 exhibit complete departure from linearity near the walls, a behaviour evidently to be associated with the presence of the boundary layers. In this respect, two points require comment. In the first place, near each wall the velocity reaches a distinct peak which tends to subside with increasing distance from the grid. Secondly, the peak near $y = h$ occurs closer to the wall than that near $y = 0$. The appearance of these velocity peaks is due to the accelerating action of the pressure drop across the grid on the fluid in the boundary layer which, by virtue of its reduced velocity, suffers a smaller change in total pressure on passing through the grid than fluid in the main stream. Since the change in total pressure diminishes progressively as the wall is approached, whereas the static pressure is constant, the velocity in the boundary layer would, in the absence of shearing stresses, increase towards the wall. This tendency is however resisted by the shearing stresses and wall friction, with the result that a maximum velocity occurs in the outer part of the boundary layer. With increasing distance from the grid, the shearing stresses smooth out the velocity peak, as can be seen in figure 5. The difference between the positions of the velocity maxima on the top and bottom walls is obviously a consequence of the difference in boundary layer thickness; for the boundary layer on the upper wall is subjected to a favourable pressure gradient whereas the pressure gradient on the lower wall is adverse, as is evident from the calculated wall pressure distribution shown in figure 7. The observed difference in boundary layer thickness on the upper and lower walls agreed quite well with a rough calculation based on Truckenbrodt's method (Schlichting 1955) using the pressure distributions of figure 7.

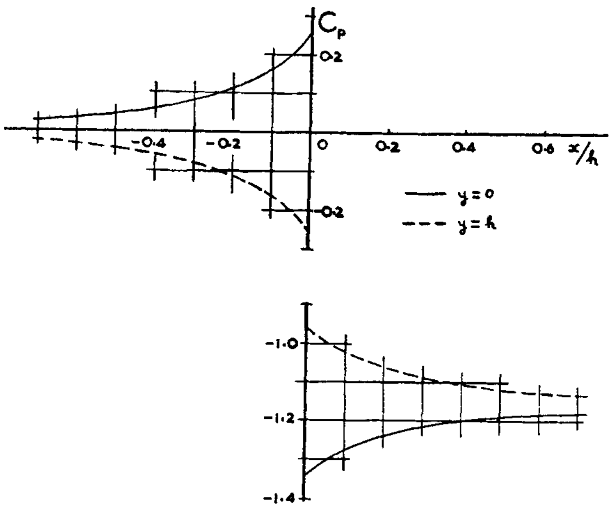

Figure 7. Calculated pressure distributions on the top and bottom walls of the tunnel; $\lambda h/U = 0{\cdot}45$, $K_0 = 1{\cdot}15$.

A final remark may be made on figure 5. Ostensibly there is no systematic change in the rate of shear between $x/h = 1{\cdot}2$ and $x/h = 4{\cdot}2$, and even the small irregularities in the velocity profile appear to suffer no appreciable decay. This is not entirely surprising, because a uniform shear flow would be expected to persist almost indefinitely if, as is likely, the turbulence generated at the grid gives rise to an approximately constant eddy viscosity. All modification to the rate of shear would then be confined to regions excluded from the action of a uniform shearing stress, that is, to the boundary layers on the tunnel walls, which only gradually encroach on the main stream.

References

Hawthorne, W. R. & Martin, M. E. 1955 *Proc. Roy. Soc.* A, **232,** 184.

Lighthill, M. J. 1956 *J. Fluid Mech.* **1,** 31.

Schlichting, H. 1955 *Boundary Layer Theory.* London : Pergamon Press.

Taylor, G. I. & Batchelor, G. K. 1949 *Quart. J. Mech. Appl. Math.* **2,** 1.

Wieghardt, K. E. G. 1953 *Aero. Quart.* **4,** 186.